

## Durham Research Online

---

### Deposited in DRO:

22 January 2016

### Version of attached file:

Published Version

### Peer-review status of attached file:

Peer-reviewed

### Citation for published item:

Cox, Steven and Garside, John and Kotsialos, Apostolos (2014) 'Control of set-up dominant multivariate manufacturing processes.', in Advances in manufacturing technology XXVIII : proceedings of the 12th International Conference on Manufacturing Research (ICMR2014) : incorporating the 29th National Conference on Manufacturing Research : 9th-11th September 2014. Southampton, UK: Southampton Solent University, pp. 209-214. International conference on manufacturing research.

### Further information on publisher's website:

<http://www.solent.ac.uk/conference-centre/events/icmr-2014/icmr-2014.aspx>

### Publisher's copyright statement:

© Southampton Solent University 2014

### Additional information:

## Use policy

---

The full-text may be used and/or reproduced, and given to third parties in any format or medium, without prior permission or charge, for personal research or study, educational, or not-for-profit purposes provided that:

- a full bibliographic reference is made to the original source
- a [link](#) is made to the metadata record in DRO
- the full-text is not changed in any way

The full-text must not be sold in any format or medium without the formal permission of the copyright holders.

Please consult the [full DRO policy](#) for further details.

## **CONTROL OF SET-UP DOMINANT MULTIVARIATE MANUFACTURING PROCESSES**

Steven Cox

School of Engineering and Computing Sciences  
Durham University  
South Road, Durham  
DH1 3LE, UK  
steven.cox@durham.ac.uk

John Garside

School of Engineering and Computing Sciences  
Durham University  
South Road, Durham  
DH1 3LE, UK  
j.a.garside@durham.ac.uk

Apostolos Kotsialos

School of Engineering and Computing Sciences  
Durham University  
South Road, Durham  
DH1 3LE, UK  
apostolos.kotsialos@durham.ac.uk

### **ABSTRACT**

A practical control chart is introduced, called multivariate Set-Up Process Algorithm (m-SUPA), which can be used to signal when a process is statistically off-target. This control chart uses a traffic light system to provide simple information to an operator about how close a measured part is to its global target. The chart works with a simple rule set resulting in process adjustments at a calculated point, rather than relying on rule-of-thumb methods. A final consideration is calculating the size of process adjustment, when one control adjustment has multiple effects on different design features. Simple feedback controllers are suggested for calculating process adjustments, providing consistency to an action taken. Simulation results suggest that m-SUPA with adjustments based on this kind of controllers is able to steer the process to a desired performance region.

**Keywords:** Multivariate, Process Control, Feedback Controllers.

### **1 INTRODUCTION**

This paper provides a framework to control multivariate manufacturing processes, which are set-up dominant in nature. A multivariate process, produces parts which have two or more correlated design features. A set-up dominant process, is a one whose dominant source of variation is between batches (Juran & Gryna 1988) and is linked to adjusting the process to the design tolerance's target during set-up. These low-volume manufacturing processes are increasingly prevalent (Julien & Holmshaw 2012).

The advancement of manufacturing technology (Shipp et al. 2012), has seen machining processes that produce parts of increasing complexity. An example from the aerospace industry is a highly complex aerofoil for a jet engine, with an excess of 25 design features produced in a single CNC machining centre. Often, in such environments a single cutting path has an effect to more than one of these design features. If a process is producing parts that are off-target in one design feature, an adjustment to a single control parameter can correct this, but it also moves other correlated design features off-target. Therefore, determining a suitable set of control adjustments that does not drive design features off-target is a critical step.

These machining centres typically make small batches of multiple part variants. With low-volume batches as small as 5-10 parts, timing of a control adjustment is, also, of critical importance. Using a rule of thumb procedure, such as measuring the first part produced then applying control parameter adjustments, has little statistical validity. Applying traditional Statistical Process Control (SPC), can lead to a batch ending production before a subgroup of parts of sufficient size for estimating the process mean position is collected. A statistically valid tool must be incorporated into the framework that can deal with both the multivariate and small batch nature of these set-up dominant processes.

Keeping design features not only within tolerance but on-target is key to ensuring that parts produced their in-service function. To measure the performance of a process for a single design feature, the capability metric  $C_{pk}$  is used.  $C_{pk}$  is defined for a process with mean ( $\mu$ ), standard deviation ( $\sigma$ ) as  $C_{pk} = \min\left[\frac{U - \mu}{3\sigma}, \frac{\mu - L}{3\sigma}\right]$  and is used to estimate the performance against the upper ( $U$ ) and lower ( $L$ ) tolerances. This measure helps defining the limits of the univariate SUPA chart, (Cox et al. 2013).

The following section describes the process control cycle. Section 3 then outlines a solution to multi feature control problems by introducing a method called multivariate Set-Up Process Algorithm (m-SUPA). This is used with a simple feedback controller to provide control adjustments. The paper concludes with some indicative simulation results.

## 2 PROCESS CONTROL FRAMEWORK

A control framework supporting a machine operator's actions with respect to process adjustments is outlined in this section. At the centre of the framework are processes used to convert a blank part into a finished and measured part. This includes the use of equipment such as CNC Machine tools and Co-ordinate Measurement Machines. The part's measured design features are then recorded as a response. If the system is set-up in the same way each time a batch of the same parts is required, the response should be identical for each part. However, external noise factors can affect the response of each part not only between batches but within a batch. Some of these noise factors are present irrespective of operator action and are seen in the process output as constant variation, i.e. as common cause variation. The operator must compensate for the remaining noise factors using the processes control parameters. It is the adjustment of these control parameters that is critically important to an effective control framework. In order for this to happen, the following steps are typically executed:

- Detecting a state of error, i.e. producing parts that are significantly off-target.
- Recommending control factor adjustments when an error state is detected.
- Implementation of adjustments by the operator.

For univariate processes, that have uncorrelated design factors with independent control parameters, which are set-up dominant, the univariate SUPA has been introduced by (Cox et al. 2013). This method provides a machine operator with a simple chart and rule set to statistically diagnose when a process is off-target. The chart is based around the tolerance of the monitored design feature and a traffic light scheme where central region around the design target is designated as the Green Zone. The regions which are between the Green Zone and the tolerance limits are the Yellow Zones. The regions outside the tolerance limits are the Red Zones. The size of the Green Zone is determined by the minimum  $C_{pk}$  required from the process, see (Cox et al. 2013) for more details.

Consecutive parts are sampled and their measured design features are categorized as Green, Yellow or Red. If a sampled part is Red it signals that the process is off-target. Two consecutive parts in the same Yellow Zone signal an off-target process. Five consecutive Green parts demonstrate the process is capable and is allowed to continue without further checks. These rules are summarised in Table 1. See also (San Matias et al. 2004) for calculating the probabilities of qualifying a capable process.

Table 1 Outline of SUPA rules for process validation and intervention.

Sampled Units	Observation	Action
1	Red Unit	Stop and Adjust
1 2	Two Consecutive Yellow Units Same Side of Target	Stop and Adjust
1 2 3 4 5	Five Consecutive Green Units	Continue Process

## 3 MULTIVARIATE SUPA

To detect if a process is off-target, the design features of parts produced are measured and recorded. These design features are organised into the design vector,  $x$ . If a single component of the design vector is outside the design tolerance, the part is scrapped.

Consider a simplified case of a part with two design features, i.e.  $x = [x_1, x_2]^T$ . If  $x_1$  and  $x_2$  have the same specified design tolerances of  $UTL = 250$  and  $LTL = 50$ , this tolerance boundary can be represented as a box, as in Figure 1(a). Let two measured parts collected with design vectors of  $x(k=1) = [200, 100]^T$  and  $x(k=2) = [40, 200]^T$ ; these points are plotted on Figure 1(a). This shows that  $x(k=1)$  is within the design tolerance and  $x(k=2)$  is outside the design tolerance. Although this information tells a user if a part is in or out of tolerance, it does not give any indication of how close a part is to the design target. However, to formulate a Green zone, and therefore an m-SUPA chart, a target  $C_{pk}$  value for each design feature needs to be defined.

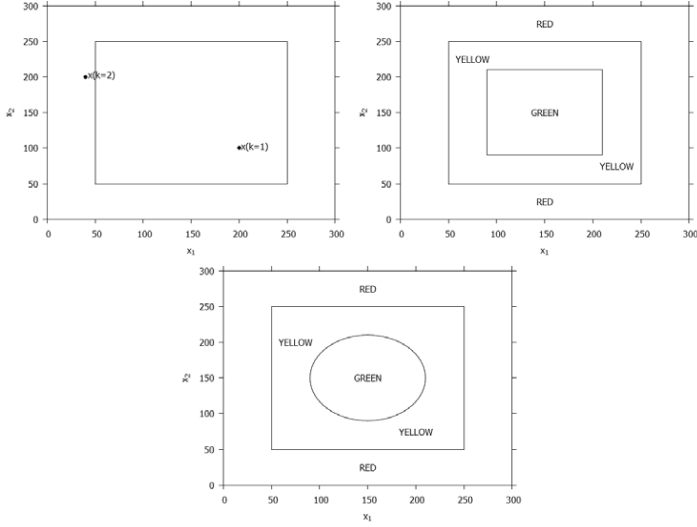


Figure 1: Plots showing: (a) tolerance boundary of  $x$ ; (b) Green zone based on univariate minimum  $C_{pk}$ ; (c) Green zone based on multivariate minimum  $C_{pk}$ .

If univariate SUPA is applied to a multi-featured part, a rectangular Green zone can be formed as in Figure 1(b). This rectangle is a percentage of each design feature tolerance boundary around its target. If the rules in Table 1 are followed with this chart, a need for process adjustment is signalled when a single dimension in the process is not conforming to the minimum required  $C_{pk}$ . However, considering the design vector in a global sense, this defines a Green zone such that the probability of an on-target process producing a Green part being greater than 94%. Therefore, if the rules of Table 1 are applied to this chart, it is less sensitive to changes in the mean position of the design vector.

A truly multivariate chart, as in Figure 1(c), is refined by determining the maximum variance for each design feature,  $s_{ii}^2$ , using the minimum required  $C_{pk}$ . This allows the definition of a covariance matrix,  $S$ , with only diagonal elements. This covariance matrix is used with the measured design vector,  $x$ , and the process target,  $T$ , to calculate a statistical distance, known as the Mahalanobis distance (Mahalanobis 1936), as follows:

$$(x - T)^T S^{-1} (x - T) < H^2. \quad (1)$$

A multidimensional Green zone is defined which is the set whose points have a Mahalanobis distance less than  $H^2$  from the target  $T$ . The left hand side of (1) has the property of following a  $\chi_{n,\alpha}^2$  distribution, where  $n$  is the degrees of freedom, which is equal to number of design features, and  $\alpha$  is the probability of a sample from a population that is on-target falling outside the Green zone.

In the case of univariate SUPA the Green zone is defined so that an on-target process has a minimum 0.94 probability of falling in the Green zone. Hence, extending this to the multivariate case will result in  $\alpha = 0.06$ . This results in a hypersphere as in Figure 1(c). Using this chart, a decision about whether a process is off-target or not, is still made by following the SUPA rules of Table 1.

Using the Mahalanobis distance in multivariate control is not a new concept. Hotelling's  $T^2$  charts have been used to provide a multivariate version of Shewhart's classic statistical process control and a multivariate version of stop-light control have been proposed (Pan 2007; Hubele 1989). However, these methods do not connect a control zone to a tangible process boundary.

#### 4 FEEDBACK CONTROLLERS

In order to model the relationship between the changes in control parameters and the resulting effect on the process output, data needs to be recorded regarding the time and size of current changes to control parameters settings. Doing this enables the calculation of a control coefficient represented by a matrix  $A$  which links the change in a control parameter,  $\Delta c$ , to its effect on the mean output of the process for each design feature. The relationship between the current process mean when produced part  $k$  has been measured,  $\bar{x}(k)$ , and the next process mean,  $\bar{x}(k+1)$ , after a control adjustment is made, is described as a linear dynamic system of the form:

$$\bar{x}(k+1) = \bar{x}(k) + A \cdot \Delta c \quad (2)$$

This assumption enables the use of a feedback controller, if the multivariate SUPA chart indicates that an adjustment to the control factors is required, to calculate the size of adjustment  $\Delta c$  (Sachs et al. 1995). In the simplified example of a part with two design features Equation (3) can be expanded as follows:

$$\bar{x}_1(k+1) = \bar{x}_1(k) + a_{11} \cdot \Delta c_1 + a_{12} \cdot \Delta c_2 \quad (3)$$

$$\bar{x}_2(k+1) = \bar{x}_2(k) + a_{21} \cdot \Delta c_1 + a_{22} \cdot \Delta c_2 \quad (4)$$

where  $a_{ij}$  are individual control coefficients from the matrix  $A$  and  $\Delta c_1$  and  $\Delta c_2$  are the adjustments of the two features process means. The values of  $a_{ij}$  are derived from historical data of the effect of control adjustments on process means. In the absence of such data, operator judgement is needed until historical data exists. Depending on the properties and the setting of the machining centre, the number of independent adjustments may vary. For example, it may be possible for the operator to make corrections for each design feature individually, in which case the dimension of the control vector  $\Delta c$  is equal to the dimension of the state vector  $\bar{x}$ . In this case, using a proportional feedback controller for  $\Delta c_1$  and  $\Delta c_2$  leads to the following calculation of the adjustments

$$\Delta c_1 = G_1[\bar{x}_1(k) - T_1], \quad \Delta c_2 = G_2[\bar{x}_2(k) - T_2] \quad (5)$$

where,  $G_1$  and  $G_2$  are gain factors and  $T_1$  and  $T_2$  are design factor target values.  $G_1$  and  $G_2$  can be set arbitrarily initially and then adjusted to improve the dynamic response when an adjustment is required, i.e. they can be fine tuned based on either on data or on simulation. It is important to note, that a change in one control factor, e.g.  $\Delta c_1$ , has an effect on both design features  $\bar{x}_1$  and  $\bar{x}_2$  through the elements of matrix  $A$ , as modelled in Equations (3) and (4). Also, these feedback control actions are only made if consecutive units fall in the Yellow or Red zones of the m-SUPA chart as per control rules of Table 1. In the following section the effect of this dynamic feedback approach is demonstrated with results from a simulation. In other words, the m-SUPA system is a combination of the discrete rules described in Table 1 and the continuous dynamic equation (2), making it a hybrid dynamical system.

#### 5 SIMULATION RESULTS

In order to test the effectiveness of this framework, a simulation model of existing processes was built. In these simulations two cases were considered. In the first one, there are two correlated design factors, and in the second there are three. In each case a process where there are as many control

parameters as features in the design vector and where there are less are examined, i.e.  $\Delta c_2 = 0$  in the 2-dimensional case and  $\Delta c_3 = 0$  in the 3-dimensional. In the simulations all design features have the same tolerance of  $UTL = 250$  and  $LTL = 50$ . Also, the Green zone covariance matrix,  $S$ , was define to validate processes with a minimum  $C_{pk} \geq 2$ .

In the first case four experiment were run with  $\bar{x}(k)$  from different initial positions of  $[100, 225]^T$ ,  $[250, 75]^T$ ,  $[150, 225]^T$  and  $[240, 210]^T$ ;  $G = [0.005, 0.005]^T$  in all cases. The actual covariance matrix of the process,  $\Sigma$ , and control coefficient remained constant throughout the simulation and are  $\Sigma = \begin{bmatrix} 100 & 90 \\ 90 & 100 \end{bmatrix}$  and  $A = \begin{bmatrix} 78 & -24 \\ 45 & 93 \end{bmatrix}$ .

The simulation experiments run to test the response of m-SUPA under these different conditions. Figure 2(a) plots the response when the system has two design features and two control parameter and 2(b) when it has one.

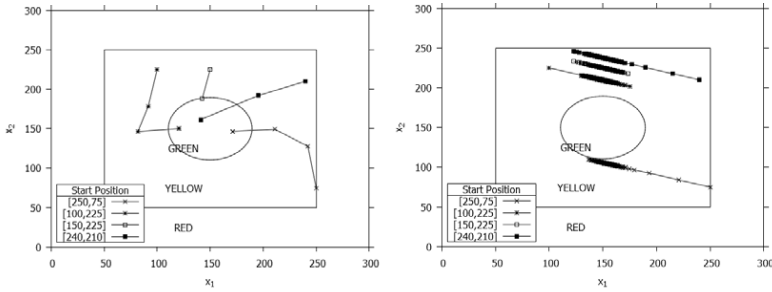


Figure 2: (a) Two design features, two control parameters; (b) two design features, two correlated control parameters.

These results show that in both situations the controller settles after a few iterations. However, in the under-defined situation shown in Figure 2(b), the process does not settle on the desired design vector target. This is due to the controller only being proportional to error of  $x_1$ ; therefore, the process tends towards settling on the  $x_1$  target of 150 irrespective of the  $x_2$  position. This is to be expected, since there is no feedback from the second design feature. This leads to additional small adjustments that are not required as the process has not settled near the centre of the Green zone, indicating that the controller is not able to drive the system in the Green zone. Designing more efficient feedback controllers for this type of task is currently under research.

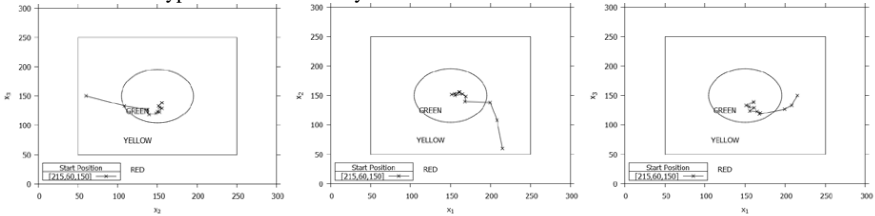


Figure 3: Three design features, three control parameters

For the second case where there are three design features, the results for the three design features and three control parameters situation are plotted in Figure 3 and for the three design features, two control parameters situation in Figure 4. The three subfigure in these two figures show the system's projected trajectory on the three planes of the system state space as measurements are taken. In both situations  $\bar{x}(k)$  starts off-target at  $[215, 60, 150]^T$ . Figure 3 shows that all design features hone in on

the target. This leads to less iterations or process adjustments. This is achieved despite the fact that a single control parameter effects all features. As expected, Figure 4 highlights the same issue that occurred in the two feature case that one design feature,  $x_3$ , does not hone in on its design target, however, the feedback controllers manage to maintain the process in the Green zone.

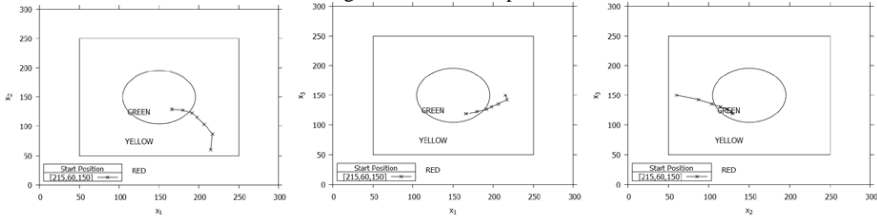


Figure 4: Three design features, two control parameters

More sophisticated control methods are going to be used for designing more efficient methods for this type of multi-dimension quality control problems.

## 6 CONCLUSIONS

This paper has presented a new framework for control of set-up dominant multivariate processes. The main advancement this method offers is ease of implementation, since the multivariate SUPA scheme clearly defines a part as Red, Yellow or Green. The simple definition allows operators to make interventions at clear times. Red parts are defined as out-of-tolerance, which provides an intuitive link for operators.

The feedback controller also provides consistent adjustments to operators, maintaining a process on target. These controllers respond particularly well when the system is fully-defined, however, in practice machine tools are typically under-defined. In this case more work needs to be done to design controllers able to make the process stay as close as possible to the target point the specifications require. Further research based on sound tools and methods from hybrid control theory is something that will take place and is expected to yield controllers that minimise the probability of qualifying off-target processes or processes that structurally have to yield some design features in the yellow zone. This work will also explore the option of re-centring Green zones on optimum positions rather than design target.

## REFERENCES

- Cox, S., Garside, J.A. & Kotsialos, A. 2013. Discrete-Event Simulation of Process Control in Low Volume High Value Industries. In *11th International Conference on Manufacturing Research*.
- Hubele, N., 1989. A multivariate and stochastic framework for statistical process control. In *Statistical Process Control in Automated..* 129–151.
- Julien, D. & Holmshaw, P. 2012. Six Sigma in a Low Volume and Complex Enviroment. *International Journal of Lean Six Sigma*, 3(1):28–44.
- Juran, J.M. & Gryna, F.M. 1988. *Quality Control Handbook*, New York: McGraw-Hill Professional.
- Mahalanobis, P., 1936. On the generalized distance in statistics. *Proceedings of the National Institute of Sciences*, 2(1)L:49–55.
- Pan, J., 2007. A study of multivariate pre-control charts. *International Journal of Production Economics*, 105(1):160–170.
- Sachs, E., Hu, A. & Ingolfsson, A., 1995. Run by run process control: combining SPC and feedback control. ... *Manufacturing, IEEE Transactions* 8(9407592):26–43.
- San Matias, S., Jabaloyes, J. & Carrion, A., 2004. Some modifications of the classical pre-control technique. *Quality and Reliability Engineering International* 20:47–60.
- Shipp, S.S. et al., 2012. *Emerging Global Trends in Advanced Manufacturing*. Nayanee Gupta Bhavya Lal.